УДК 519.6 doi: 10.20998/2220-4784.2021.01.06

N. V. CHEREMSKAYA

APPLICATION OF THE CORRELATION THEORY OF INHOMOGENEOUS RANDOM FIELDS TO THE STUDY OF THE STATISTICALLY INHOMOGENEOUS SCREEN MODEL

The article considers the problem of finding a field created by a system of fluctuating sources on the screen, which is characterized by a correlation function, where $\widetilde{K_{AA}\left(\vec{r}_1,\vec{r}_2\right)}$ the correlation function is separable. This image corresponds to a random field on the screen, which is the sum of a separable field and a heterogeneous random field of the first rank, which significantly changes the correlation radius at a distance *l.* The model studied in this paper does not assume uncorrelated sources and coincidence of laws of intensity change and therefore corresponds to a system of sources with significantly different intensities and laws of their change in the direction of wave propagation in the transverse plane. The correlation function of the sources be not assumed to be separable and the field distribution on the screen is an inhomogeneous random field of the first rank or is the sum of a separable field and a statistically inhomogeneous field of the first rank. To find a solution in the approximation of a parabolic equation, a method of immersion in the corresponding Hilbert space is proposed, which allows one to quickly and efficiently find the statistical characteristics of the solution. As an example, the influence of statistical inhomogeneity on the intensity function of a luminous screen, which has the shape of a round disk, is considered. An off-screen correlation function is obtained, which contains information on the size and nature of inhomogeneities of emitting sources on a luminous screen. A numerical analysis of the representation for the correlation function is carried out in the case when the statistical heterogeneous of the environmen is generated by the presence of a continuous spectrum or a spectrum at zero. The article obtains approximate calculation formulas for the average temperature field and its dispersion, which take into account fluctuation processes in the calculation of thermal regimes of solar panels, which allow to make appropriate corrections in theoretical calculations.

Key words: modeling of statistical properties of the medium, correlation function, separable field, statistical inhomogeneity, Hilbert space, scalar product, continuous spectrum of the operator.

Introduction.

In the modern theory of the propagation of electromagnetic and sound waves in the atmosphere in many cases we have to pay attention to turbulence, which causes fluctuations in the refractive index of air. In some cases, the turbulence of the atmosphere causes fluctuations in the parameters of the waves that propagate through it (amplitudes, directions of propagation, frequencies, phases, etc.). These effects are sources of distortions and errors in communication systems, location, radio navigation, control systems. Fluctuations in the parameters of light waves are especially influential, which is now becoming especially important in connection with the development of optical quantum generators [1–4].

In modeling the statistical properties of the environment (atmosphere, ocean), it is usually assumed that these properties can be described by a homogeneous and isotropic field or a random field with homogeneous increments of the first order. The structure of the corresponding correlation function is determined on the basis of the solution of the equation of homogeneous and isotropic turbulence, which can be obtained by averaging the equation of hydrodynamics using any hypothesis of closure [2, 3]. Fluctuations in fluid's velocity (wind's) and temperature in a random medium lead to corresponding fluctuations in pressure or refractive index (dielectric constant). Therefore, the task of propagating sound or electromagnetic waves becomes stochastic. From a mathematical point of view, the analysis of wave propagation in a random medium is reduced to the solution of a wave equation (vector or scalar) with random coefficients. Within correlation theory and in terms of applications (in particular, in the theory of wave propagation in random media [4]), the main objects of theoretical research are the mathematical expectation and the correlation function of the solution of the wave equation. The so-called "dishonest" method is used to solve this problem. This method consists in the fact that randomness is used in direct averaging of equations with random parameters, which after averaging are not closed and require additional unproven assumptions about special statistical properties of the solution for closure. These assumptions greatly simplify the problem and allow it to be solved explicitly. Many of the results obtained by the "dishonest" method agree quite well with the experimental data, which can serve as a justification for the probability of assuming one or another statistical property of the solution [5]. However, models that use a homogeneous and isotropic field or a random field with homogeneous first-order increments are not suitable for describing transient media (for example, plasma randomly changes its charge) or when electromagnetic waves propagate near the globe and statistical inhomogeneity environment is disturbed, as well as the scattering of electromagnetic waves in the wake of the rocket, the scattering of waves in the atmosphere of Venus and other planets of the solar system. Solutions to these problems require the rejection of the use of the correlation theory of stationary random functions or homogeneous random fields and the involvement of such models of correlation functions that would take into account statistical nonstationarity or inhomogeneity. This article discusses the problem of finding a field created by a system of fluctuating sources on the screen. Let the correlation function of the sources not be assumed to be separable and the field distribution on the screen be an inhomogeneous random field of the first rank or be the sum of the separable field and the statistically inhomogeneous field of the first rank. To find the solution in the parabolic equation approximation, a method of immersion in the corresponding Hilbert space is proposed, which allows to quickly and efficiently find the statistical characteristics of the solution.

As an example, the influence of statistical inhomogeneity on the function of the intensity of the screen, which glows and has the shape of a round disk, is considered. An off-screen correlation function is obtained, which contains information about the size and nature of the inhomogeneity of the emitting sources on the illuminated screen. Next, the fluctuations of the phase and amplitude of a plane wave propagating in a statistically inhomogeneous atmosphere are analyzed by the method of smooth perturbations.

Problem statement in general and its connection with important scientific or practical tasks.

Let's analyze the passage of a wave through a random medium or a layer of such a medium. Assuming that the thickness of the medium or layer is small enough, for example, a glowing screen, you can use the approximation of a thin screen, given that the field outside the screen is created by a system of fluctuating sources contained in the plane. The probabilistic properties of the sources are assumed to be known. In the approximation of the parabolic equation for the complex amplitude $A\left(\vec{r},z\right)$ (the z-axis is directed along the direction of wave propagation) we obtain the Cauchy problem

$$\left(\frac{\partial}{\partial z}+i\frac{1}{2k}\Delta_{\perp}\right)A\left(\vec{r},z\right)=0,\quad A\big|_{z=0}=A_0\left(\vec{r}\right),\qquad(1)$$

where $\vec{r}=\left(x,y\right)$, and $A_0\left(\vec{r}\right)$ random field with $MA_0\left(\vec{r}\right)\equiv 0$ and a known correlation function. This problem in the case when the field in the plane z = 0 is statistically homogeneous was investigated in [6, 7]. However, of practical interest are problems when the field $A_0\left(\vec{r}\right)$ is statistically inhomogeneous.

The case where the correlation function of the field $A_0\left(\vec{r}\right)$ has the form $K_{A_0A_0}\left(\vec{r_1},\vec{r_2}\right)=K_0\left(\frac{\vec{r_1}+\vec{r_2}}{2}\right)K\left(\vec{r_2}-\vec{r_1}\right)$ (separable correlation function) was studied in detail in [8, 9, 10, 11], where a model of the turbulent atmosphere of Venus was constructed. The separability of the correlation function is used $l_{A_0}\ll L$ when де L – the characteristic scale of the variance of the field variance, the mean field and the correlation coefficient of the field by argument $\frac{\vec{r_1}+\vec{r_2}}{2}$, and the l_{A_0} – radius of correlation of the field by argument $\vec{r_2}-\vec{r_1}$, i.e. the statistical characteristics of the field change smoothly. Thus, the correlation function of a homogeneous field is modeled by a function that changes slowly.

Teaching the main research material.

This article continues the study of the statistically inhomogeneous screen model that is affected this article will consider a more general model of a statistically inhomogeneous screen, which is characterized by a correlation function of the form

$$K_{A_0A_0}\left(\vec{r_1},\vec{r_2}\right)=\widetilde{K_{AA}}\left(\vec{r_1},\vec{r_2}\right)+$$
$$+\int\int\int_0^{\infty}\varphi\left(\vec{r_1}+\vec{\tau}\right)\overline{\varphi\left(\vec{r_2}+\vec{\tau}\right)}dV_{\tau}\qquad(2),$$

where $\widetilde{K_{AA}}\left(\vec{r_1},\vec{r_2}\right)$ separable correlation function. This image corresponds to a random field on the screen, which is the sum of a separable field and a heterogeneous random field of the first rank, which significantly changes the correlation radius at a distance l_{A_0}. To clarify the physical content of the separability of the correlation function and to clarify the physical capabilities of models of statistically inhomogeneous fields $A_0\left(\vec{r}\right)$ studied in the article, consider a random field created by a system of uncorrelated statistically inhomogeneous sources:

$$A_0\left(\vec{r}\right)=\sum_{k=1}^{n}A_k\left(\vec{r}\right),\qquad A_k\left(\vec{r}\right)=a_k\left(\vec{r}\right)\xi_k\left(\vec{r}\right),\qquad(3)$$

where $a_k\left(\vec{r}\right)$ deterministic functions, $\xi_k\left(\vec{r}\right)$ uncorrelated statistically inhomogeneous fields of the first rank [12].

Suppose that the sources first have the same law of decline, i.e. $a_k\left(\vec{r}\right)=C_ke^{\vec{\alpha}\vec{r}}$, and $\xi_k\left(\vec{r}\right)$

$$K_{A_0A_0}\left(\vec{r_1},\vec{r_2}\right)=F\left(\frac{\vec{r_1}+\vec{r_2}}{2}\right)\widetilde{K_{A_0A_0}}\left(\vec{r_2}-\vec{r_1}\right),\qquad(4)$$

that is, the correlation function satisfies the condition of separability.

Thus, the separability of the correlation function corresponds not only to the smoothness of spatial or temporal change of statistical inhomogeneities on the correlation scale, but also means the noncorrelation of statistically inhomogeneous fields that have the same laws of change in species intensity.

Model (2), which is studied in this article, does not assume uncorrelated sources and coincidence of laws of intensity change and therefore corresponds to a system of sources with significantly different intensities and laws of their change in the direction of wave propagation in the transverse plane. We return to (1) and invest $A_0\left(\vec{r}\right)$ in Hilbert space H_{A_0}, and then we get an auxiliary problem in Hilbert space

$$\left(\frac{\partial}{\partial z}+i\frac{1}{2k}\Delta_f\right)\widehat{A\left(\vec{r},z\right)}=0,\quad \widehat{A}\Big|_{z=0}=\widehat{A_0\left(\vec{r}\right)},\qquad(5)$$

$$K_{AA}\left(\vec{r_1},\vec{r_2},z\right)=\left\langle \widehat{A\left(\vec{r_1},z\right)},\widehat{A\left(\vec{r_2},z\right)}\right\rangle_{H_{A_0}}, \quad (6)$$

where parentheses denote $\langle\cdot,\cdot\rangle$ a scalar product in Hilbert space H_{A_0}.

Consider the case of the evolutionarily represented field $\widehat{A_0\left(\vec{r}\right)}=e^{ixB_1+iyB_2}f_0$, $f_0\in H_{A_0}$, and $\left[B_1,B_2\right]=0$. Then the solution of the Cauchy problem in this case takes the form

$$\widehat{A\left(\vec{r},z\right)}=e^{\frac{i}{2k}\left(B_1^2+B_2^2\right)z+ixB_1+iyB_2}f_0. \quad (7)$$

Then for the transverse correlation function

$$K_{AA}\left(\vec{r_1},\vec{r_2},z\right)=$$
$$=\left\langle e^{\frac{i}{2k}\left(B_1^2+B_2^2\right)z+ix_1B_1+iy_1B_2}f_0,e^{\frac{i}{2k}\left(B_1^2+B_2^2\right)z+ix_2B_1+iy_2B_2}f_0\right\rangle. \quad (8)$$

From (8) it is seen that for a statistically homogeneous field $\left(B_1=B_1^*,B_2=B_2^*\right)$ the transverse correlation function does not depend on *z*. If the solution of the parabolic equation is bounded for all x_1 and y_1, then the operators B_1 and B_2 are similar to self-adjoint

$$K_{AA}\left(\vec{r_1},\vec{r_2},z\right)=$$
$$=\left\langle C^*Ce^{\frac{i}{2k}\left(\tilde{B}_1^2+\tilde{B}_2^2\right)z+ix_1\tilde{B}_1+iy_1\tilde{B}_2}\tilde{f}_0,e^{\frac{i}{2k}\left(\tilde{B}_1^2+\tilde{B}_2^2\right)z+ix_2\tilde{B}_1+iy_2\tilde{B}_2}\tilde{f}_0\right\rangle, \quad (9)$$

where $\tilde{B}_j$ self-adjoint operators, and *C* is the operator that performs the similarity.

In the future, for simplicity, we limit ourselves to the case when и $A_0\left(\vec{r}\right)=A_0\left(x\right)$, i.e., does not depend on *y*. Let $A_0\left(x\right)\equiv 0$ at $x<0$ and $A_0\left(x\right)=e^{iB_1x}f_0$ at $x\ge 0$.. Using (8) to $K_{AA}\left(x_1,x_2,z\right)$ obtain the expression

$$K_{AA}\left(x_1,x_2,z\right)=$$
$$=\frac{k}{4z}\int_0^\infty\int_0^\infty e^{-\frac{ik}{2z}\left[\left(x_1-\xi\right)^2-\left(x_2-\eta\right)^2\right]}\left\langle e^{i\xi B_1}f_0,e^{i\eta B_1}f_0\right\rangle d\xi d\eta. \quad (10)$$

The expression for $K_{AA}\left(x_1,x_2,z\right)$ is obtained in the case when the model considered in [12] is used.

$$K_{AA}\left(x_1,x_2,z\right)=$$
$$=\frac{k}{4z}\int_0^\infty\int_0^\infty e^{-\frac{ik}{2z}\left[\left(x_1-u\right)^2-\left(x_2-v\right)^2\right]}\frac{1}{i}\sum_{l,m=1}^{p}b_l\overline{b_m}\frac{e^{i\lambda_lu-i\overline{\lambda}_mv}}{\lambda_l-\overline{\lambda_m}}dudv,$$

or if we limit ourselves to the real part of the model correlation function

$$K_{AA}\left(x_1,x_2,z\right)=\frac{k}{4z}\int_0^\infty\int_0^\infty e^{-\frac{ik}{2z}\left[\left(x_1-u\right)^2-\left(x_2-v\right)^2\right]}\sum_{l,m=1}^{p}b_lb_me^{-\frac{\beta_l^2u+\beta_m^2v}{2}}\times$$
$$\times\frac{\left[\left(\alpha_l-\alpha_m\right)\sin\left(\alpha_lu-\alpha_mv\right)-\frac{\beta_l^2+\beta_m^2}{2}\cos\left(\alpha_lu-\alpha_mv\right)\right]}{\left(\alpha_l-\alpha_m\right)^2+\frac{\left(\beta_l^2+\beta_m^2\right)^2}{4}}dudv=$$
$$=\frac{k}{4z}\sum_{l,m=1}^{p}\frac{b_lb_m}{\left(\alpha_l-\alpha_m\right)^2+\frac{\left(\beta_l^2+\beta_m^2\right)^2}{4}}\times$$
$$\times[\left(\alpha_l-\alpha_m\right)\left(\psi_{1l}\left(z,x_1\right)\psi_{2m}\left(z,x_2\right)-\psi_{2l}\left(z,x_1\right)\psi_{1m}\left(z,x_2\right)\right)-$$
$$-\frac{\beta_l^2+\beta_m^2}{2}\left(\psi_{2l}\left(z,x_1\right)\psi_{2m}\left(z,x_2\right)-\psi_{1l}\left(z,x_1\right)\psi_{2l}\left(z,x_2\right)\right)],$$

$$\text{Where } \psi_{1l}\left(z,x\right)=\int_0^\infty e^{-\frac{ik}{2z}\left(x_1-u\right)^2-\frac{\beta_l^2u}{2}}\sin\alpha_ludu,$$

$$\psi_{2l}\left(z,x\right)=\int_0^\infty e^{-\frac{ik}{2z}\left(x_1-u\right)^2-\frac{\beta_l^2u}{2}}\cos\alpha_ludu.$$

$$\psi\left(z,x\right)=e^{-\frac{ik}{2z}x^2}\sqrt{\frac{2z}{k}}\int_0^\infty e^{-\sqrt{\frac{2z}{k}}av}\begin{Bmatrix}\cos v^2\\ \sin v^2\end{Bmatrix}dv,$$

where $k,z>0$, $a=\frac{\beta_l^2}{2}-i\left(\pm\alpha_l-\frac{kx}{z}\right)$, $\operatorname{Re}a>0$.

$$\int_0^\infty e^{-\sqrt{\frac{2z}{k}}av}\cos v^2dv=\left[\frac{1}{2}-S\left(\sqrt{\frac{z}{\pi k}}a\right)\right]\cos\frac{a^2z}{2k}-$$
$$-\sqrt{\frac{\pi}{2}}\left[\frac{1}{2}-C\left(\sqrt{\frac{z}{\pi k}}a\right)\right]\sin\frac{a^2z}{2k},$$
$$\int_0^\infty e^{-\sqrt{\frac{2z}{k}}av}\sin v^2dv=\left[\frac{1}{2}-C\left(\sqrt{\frac{z}{\pi k}}a\right)\right]\cos\frac{a^2z}{2k}-$$
$$-\sqrt{\frac{\pi}{2}}\left[\frac{1}{2}-S\left(\sqrt{\frac{z}{\pi k}}a\right)\right]\sin\frac{a^2z}{2k},$$

where $C\left(y\right)=\int_0^y\cos\left(\frac{\pi}{2}t\right)^2dt$, $S\left(y\right)=\int_0^y\sin\left(\frac{\pi}{2}t\right)^2dt$ Fresnel integrals, and $y=u+iv$ [14, 15, 16, 17, 18].

Because of $\lim_{y\to\infty}S\left(y\right)=\lim_{y\to\infty}C\left(y\right)=\frac{1}{2}$, so $\lim_{z\to\infty}K\left(x_1,x_2,z\right)=0$.

In the case where B_1 the Voltaire operator, and f_0 coincides with the channel element of the operator B_1, the correlation function $A\left(x,z\right)$ takes the form

$$K_{AA}(x_1,x_2,z)=\int_0^l \Phi(z,x_1,u)\overline{\Phi(z,x_2,u)}du\,, \qquad (11)$$

where $\Phi(z,x_1,u)=\sqrt{\dfrac{ik}{z}}\int_0^\infty e^{-\frac{ik}{z}(x-x_1)^2}J_0\left(2\sqrt{x_1(l-u)}\right)dx_1$. (12)

These images for the correlation function of the complex amplitude can be used, in particular, to build models of various statistically inhomogeneous screens.

As an example, consider the effect of statistical inhomogeneity on the degree of spatial incoherence of the source, which has the shape of a disk of radius *a*. To do this, consider the diffraction of a wave at a circular hole of radius *a* centered at point *O* in the case where the correlation function of such a screen takes the form

$$K_{A_0A_0}(\vec{r}_1,\vec{r}_2)=K_{\perp 0}(\vec{r},\vec{R})+\left\langle e^{i\vec{r}_1\vec{B}_1}f_0,e^{i\vec{r}_2\vec{B}_1}f_0\right\rangle, \qquad (13)$$

where $K_{\perp 0}(\vec{r},\vec{R})=I(\vec{R})\delta(\vec{r})$, $I(\vec{R})=\begin{cases}I_0, R\le a\\ 0, R\ge a\end{cases}$,

$\vec{r}=\vec{r}_1-\vec{r}_2$, $\vec{R}=\dfrac{\vec{r}_1+\vec{r}_2}{2}$, that is $A_0(\vec{r})=A_{0\perp}(\vec{r})+\tilde{A}_0(\vec{r})$,

where $A_{0\perp}(\vec{r})$ and $\tilde{A}_0(\vec{r})$ uncorrelated, $A_0(\vec{r})$ statistically inhomogeneous field of the first rank.

This type of correlation function corresponds, for example, to either take into account the influence of the penumbra on the structure of the random field on the screen, or to take into account statistical inequalities of the screen edge, or the presence of statistical inhomogeneity of the emitter system.

Then at a distance z on the plane of the screen $\vec{r}_1=0$ and $\vec{r}_2(s,0)\ne 0$ at and for $K_{AA}(\vec{r}_1,\vec{r}_2)$ we have

$$K_{AA}(\vec{r}_1,\vec{r}_2,z)=I_0\frac{kae^{\frac{iks^2}{z}}}{2\pi zs}J_1\left(\frac{kas}{z}\right)+$$
$$+\frac{k}{4z}\int_0^\infty\int_0^\infty e^{-\frac{ik}{2z}\left[u^2-(s-v)^2\right]}\left\langle e^{iuB_1}f_0,e^{ivB_1}f_0\right\rangle dudv \qquad (14)$$

In the simplest case, when B_1 it acts as a multiplication operator for a complex number

$\lambda_0=\alpha_0+\dfrac{i\beta_0^2}{2}$, we have $K_{AA}(\vec{r}_1,\vec{r}_2,z)=$

$$=I_0\frac{kae^{\frac{iks^2}{z}}}{2\pi zs}J_1\left(\frac{kas}{z}\right)+\Psi(0,z)\overline{\Psi(s,z)}\,, \qquad (15)$$

where $\Psi(s,z)=\sqrt{\dfrac{k}{z}}\|f_0\|\int_0^\infty e^{-\frac{ik}{2z}(s-u)^2+iu\lambda_0}du$. (16)

In the case when the operator B_1 has a spectrum at zero, $K_{AA}(\vec{r}_1,\vec{r}_2)$ we obtain the expression

$$K_{AA}(\vec{r}_1,\vec{r}_2,z)=I_0\frac{kae^{\frac{iks^2}{z}}}{2\pi zs}J_1\left(\frac{kas}{z}\right)+$$
$$+\int_0^l \Phi(z,0,u)\overline{\Phi(z,s,u)}du\,, \qquad (17)$$

where

$$\Phi(z,s,u)=\sqrt{\frac{ik}{2\pi z}}\int_0^\infty e^{-\frac{ik}{z}(s-x_1)^2}J_0\left(2\sqrt{x_1(l-u)}\right)dx_1 \qquad (18)$$

Thus, it is shown that in the correlation function $K_{A_0A_0}(\vec{r}_1,\vec{r}_2,z)$ together with the screen sizes „*a*", which correspond, for example, to the linear size of the star, there is information about the scale of statistical inhomogeneity of the sources contained in the screen plane. In this case, as can be seen from (15) and (16), against the background of the usual correlation function, additional oscillations appear with a scale that corresponds to the scale of inhomogeneities of the sources.

This indicates the possibility of using a known correlation function to determine not only the linear dimensions of the screen (stars), but also the scale of heterogeneity of the sources that make it up. In the case of a zero spectrum, the correlation function $K_{A_0A_0}(\vec{r}_1,\vec{r}_2)$ contains information about the intensity of the brightly localized spot on the glowing screen.

Consider the fluctuations of the amplitude of a plane wave that propagates in a turbulent atmosphere statistically inhomogeneous in the direction x and statistically homogeneous in the directions y and z. Then, for the complex phase in the approximation of the smooth perturbation method, we obtain the equation [4]

$$\frac{\partial^2\Phi_1}{\partial y^2}+\frac{\partial^2\Phi_1}{\partial z^2}+2ik\frac{\partial\Phi_1}{\partial x}=-k^2\varepsilon_1(x,y,z)\,, \qquad (19)$$

where $\varepsilon_1(x,y,z)$ is a random dielectric constant that describes a random inhomogeneous medium. After using the spectral schedules for Φ_1 and ε_1 we obtain

$$\varepsilon_1(x,y,z)=\varepsilon_1(x,0,0)+$$
$$+\int_{-\infty}^{\infty}\int(e^{i(\chi_2y+\chi_3z)}-1)u_\varepsilon(d\chi_2,d\chi_3,x),$$
$$\Phi_1(x,y,z)=\Phi_1(x,0,0)+$$
$$+\int_{-\infty}^{\infty}\int(e^{i(\chi_2y+\chi_3z)}-1)u_\Phi(d\chi_2,d\chi_3,x),$$

for u_Φ we obtain the equation

$$2ik\frac{\partial u_\Phi}{\partial x}-\chi^2 u_\Phi=-k^2 u_\varepsilon\left(d\chi_2,d\chi_3,x\right),$$

where $\chi^2=\chi_2{}^2+\chi_3{}^2$. The solution of which satisfies the boundary condition $u_\Phi\left(d\chi_2,d\chi_3,0\right)=0$ and has the form

$$u_\Phi\left(d\chi_2,d\chi_3,x\right)=\frac{ik}{2}\int_0^x \exp\left(-\frac{i\chi^2\left(x-x'\right)}{2k}\right)u_\varepsilon\left(d\chi_2,d\chi_3,x'\right)dx'.$$

For the average value

$$Mu_\Phi\left(d\chi_2,d\chi_3,x\right)\overline{u_\Phi\left(d\chi_2',d\chi_3',x'\right)}$$

we get the image

$$\begin{aligned}&Mu_\Phi\left(d\chi_2,d\chi_3,x\right)\overline{u_\Phi\left(d\chi_2',d\chi_3',x'\right)}=\\&=\delta\left(\chi_2-\chi_2'\right)\delta\left(\chi_3-\chi_3'\right)d\chi_2,d\chi_3 d\chi_2',d\chi'\times\\&\times\frac{k^2}{4}\int_0^x\int_0^x e^{\frac{i\chi^2}{2k}\left(x'-x''\right)}F_\varepsilon\left(\chi_2,\chi_3,x',x''\right)dx'dx''.\end{aligned}$$

Suppose for simplicity that

$$F_\varepsilon\left(\chi_2,\chi_3,x',x''\right)=F_\varepsilon\left(\chi_2,\chi_3\right)F\left(x',x''\right),$$

and the structure $F\left(x',x''\right)$ takes the form [12].

$$\begin{aligned}&F\left(x',x''\right)=\int_0^\infty \varphi\left(x'+\tau\right)\varphi\left(x''+\tau\right)d\tau=\\&=\int_0^\infty\left[u\left(x'+\tau\right)u\left(x''+\tau\right)+v\left(x'+\tau\right)v\left(x''+\tau\right)\right]d\tau.\end{aligned}$$

It is this structure F that takes into account the statistical heterogeneity along the direction x. Then

$$\begin{aligned}&Mu_\Phi\left(d\chi_2,d\chi_3,x\right)\overline{u_\Phi\left(d\chi_2',d\chi_3',x'\right)}=\\&=\delta\left(\chi_2-\chi_2'\right)\delta\left(\chi_3-\chi_3'\right)\times\\&\quad\times F\left(\chi_2,\chi_3\right)d\chi_2,d\chi_3 d\chi_2',d\chi'\times\\&\quad\times\int_0^\infty\left[\left|\Phi\left(x,\tau\right)\right|^2+\left|\Psi\left(x,\tau\right)\right|^2\right]d\tau,\end{aligned}\tag{20}$$

where

$$\Phi\left(x,\tau\right)=\int_0^x e^{\frac{i\chi^2}{2k}x'}u(x'+\tau)dx',$$

$$\Psi\left(x,\tau\right)=\int_0^x e^{\frac{i\chi^2}{2k}x'}v(x'+\tau)dx'.\tag{21}$$

Images (20), (21) can be used to model statistical inhomogeneities in the axis direction x. Consider the case of a statistically inhomogeneous system characterized by a discrete spectrum. Using the results [12], for $\Phi\left(x,\tau\right)$ we obtain.

$$\begin{aligned}\Phi\left(x,\tau\right)&=\int_0^x e^{\frac{i\chi^2}{2k}x'}C_1 e^{-\frac{\beta_1^2}{2}(x'+\tau)}\cos\alpha_1(x'+\tau)dx'=\\&=e^{\frac{i\chi^2}{2k}\tau}\int_\tau^{x+\tau}e^{\gamma y}\cos\alpha_1 y dy=\\&=e^{\frac{i\chi^2}{2k}\tau}\{\frac{\alpha_1}{\alpha_1^2+\gamma^2}e^{\gamma\tau}\left(e^{\gamma x}\sin\alpha_1\left(x+\tau\right)-\sin\alpha_1\tau\right)+\frac{\gamma}{\alpha_1^2+\gamma^2}e^{\gamma\tau}\times\\&\quad\left(e^{\gamma x}\cos\alpha_1\left(x+\tau\right)-\cos\alpha_1\tau\right)\},\end{aligned}$$

$$\begin{aligned}\Psi\left(x,\tau\right)&=\int_0^x e^{\frac{i\chi^2}{2k}x'}C_1 e^{-\frac{\beta_1^2}{2}(x'+\tau)}\sin\alpha_1(x'+\tau)dx'=\\&=e^{\frac{i\chi^2}{2k}\tau}\int_\tau^{x+\tau}e^{\gamma y}\sin\alpha_1 y dy=\\&=e^{\frac{i\chi^2}{2k}\tau}\{\frac{\alpha_1}{\alpha_1^2+\gamma^2}e^{\gamma\tau}\left(-e^{\gamma x}\cos\alpha_1\left(x+\tau\right)+\cos\alpha_1\tau\right)+\\&+\frac{\gamma}{\alpha_1^2+\gamma^2}e^{\gamma\tau}\left(e^{\gamma x}\sin\alpha_1\left(x+\tau\right)-\sin\alpha_1\tau\right)\},\end{aligned}$$

where $\gamma=-\frac{\beta_1^2}{2}+\frac{i\chi^2}{2k}$.

Substituting these expressions in (20), we can obtain images for the correlation function.

Conclusions and prospects for further development of this area.

The models of nonstationary functions obtained in the article allow us to take into account the statistical inhomogeneity of the medium, for example, when studying the propagation of electromagnetic waves near the boundary of a random medium.

Model images for the correlation function can be obtained for partial cases of the spectrum of nonstationary random functions, and the corresponding real-value correlation functions can be constructed, which contain information about the complex spectrum.

This type of correlation function corresponds; for example, to either take into account the influence of the penumbra on the structure of the random field on the screen, or to take into account statistical inequalities of the screen edge, or the presence of statistical inhomogeneity of the emitter system. These images for the correlation function of the complex amplitude can be used, in particular, to build models of various statistically inhomogeneous screens.

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Надійшла (received) 19.05.2021

Відомості про авторів / Сведения об авторах / About the Authors

Черемська Надія Валентинівна (Черемская Надежда Валентиновна, Cheremskaya Nadezhda Valentinovna) – кандидат технічних наук, доцент, Національний технічний університет «Харківський політехнічний інститут», м. Харків, Україна; тел.: (050) 225-15-44; e-mail: cheremskaya66@gmail.com.

Н. В. ЧЕРЕМСЬКА

ЗАСТОСУВАННЯ КОРЕЛЯЦІЙНОЇ ТЕОРІЇ НЕОДНОРІДНИХ ВИПАДКОВИХ ПОЛІВ ДО ДОСЛІДЖЕННЯ МОДЕЛІ СТАТИСТИЧНО НЕОДНОРІДНОГО ЕКРАНА

У статті розглянута задача про знаходження поля, яке створюється системою флуктуючих джерел, що знаходяться на екрані, яка характеризується кореляційною функцією, де $\widetilde{K_{AA}\left(\vec{r}_1,\vec{r}_2\right)}$ сепарабельна кореляційна функція. Це зображення відповідає випадковому полю на екрані, що є сумою сепарабельного поля та неоднорідного випадкового поля першого рангу, у якого істотно змінюється радіус кореляції на відстані *l*. Модель, яка вивчається в цій статті, не припускає некорельованості джерел та збігу законів змінювання інтенсивностей та тому відповідає системі джерел з істотно відмінними інтенсивностями та законами їхнього змінювання в напрямку поширення хвилі в поперечний площині. Кореляційна функція джерел не припускається сепарабельною та розподіл поля на екрані є неоднорідним випадковим полем першого рангу або є сумою сепарабельного поля та статистично неоднорідного поля першого рангу. Для знаходження розв'язку в наближенні параболічного рівняння запропоновано метод занурення у відповідний гільбертів простір, який дозволяє швидко та ефективно відшукувати статистичні характеристики розв'язку. Як приклад розглянуто вплив статистичної неоднорідності на функцію інтенсивності екрану, який світиться та має форму круглого диску. Отримана кореляційна функція поза екраном, яка містить інформацію про розмір та характер неоднорідностей випромінюючих джерел на екрані, що світиться. Проведено чисельний аналіз зображення для кореляційної функції у випадку, коли статистична неоднорідність середовища породжується наявністю неперервного спектру або спектру в нулі. У статті одержано наближені розрахункові формули для середнього температурного поля та його дисперсії, що враховують флуктуаційні процеси при розрахунку теплових режимів сонячних батарей, які дозволяють внести відповідні поправки при теоретичних розрахунках.

Ключові слова: моделювання статистичних властивостей середовища, кореляційна функція, сепарабельне поле, статистична неоднорідність, гільбертів простір, скалярний добуток, неперервний спектр оператора.

Н. В. ЧЕРЕМСКАЯ

ПРИЛОЖЕНИЕ КОРРЕЛЯЦИОННОЙ ТЕОРИИ НЕОДНОРОДНЫХ СЛУЧАЙНЫХ ПОЛЕЙ К ИССЛЕДОВАНИЮ МОДЕЛИ СТАТИСТИЧЕСКИ НЕОДНОРОДНОГО ЭКРАНА

В статье рассмотрена задача о нахождении поля, создаваемого системой флуктуирующих источников, находящихся на экране, которое характеризуется корреляционной функцией, где $\widetilde{K_{AA}\left(\vec{r}_1,\vec{r}_2\right)}$ сепарабельная корреляционная функция. Это представление соответствует случайному полю на экране, являющегося суммой сепарабельного поля и неоднородного случайного поля первого ранга, у которого существенно меняется радиус корреляции на расстоянии *l*. Модель, которая изучается в этой статье, не предполагает некоррелируемости источников и совпадения законов изменения интенсивностей и поэтому соответствует системе источников с существенно отличающимися интенсивностями и законами их изменения в направлении распространения волны поширення хвилі в поперечной плоскости. Корреляционная функция источников не предполагается сепарабельнойльною и распределение поля на экране является неоднородным случайным полем первого ранга или является суммой сепарабельного поля и статистически неоднородного поля первого ранга. Для нахождения решения в приближении параболического уравнения предложен метод погружения в соответствующее гильбертово пространство, который позволяет быстро и эффективно находить статистические характеристики решения. В качестве примера рассмотрено влияние статистической неоднородности на функцию интенсивности светящегося экрана, который имеет форму круглого диска. Получена корреляционная функция вне экрана, содержащая информацию о размере и характере неоднородностей излучающих источников на светящемся экране. Проведен численный анализ представления для корреляционной функции в случае, когда статистическая неоднородность среды порождается наличием непрерывного спектра или спектра в нуле. В статье получены приближенные расчетные формулы для среднего температурного поля и его дисперсии, которые учитывают флуктуационные процессы при расчете тепловых режимов солнечных батарей, позволяющие внести соответствующие поправки при теоретических расчетах.

Ключевые слова: моделирование статистических свойств среды, корреляционная функция, сепарабельное поле, статистическая неоднородность, гильбертово пространство, скалярное произведение, непрерывный спектр оператора.