УДК 519.6 doi: 10.20998/2220-4784.2024.01.04

N. V. CHEREMSKA

MODELS OF CORRELATION FUNCTIONS OF NONSTATIONARY PROCESSES AND SEQUENCES FOR TECHNOLOGICAL SYSTEMS

Precision technological processes for the production of modern microelectronic products require compliance with the quality of source materials, working environments, and precise adherence to regimes. Due to natural fluctuations in the properties of materials and the environment, and variable states of all technological processes, the parameters of the product and technological processes cannot be described by deterministic laws. Due to the inevitable natural properties of fluctuations in the parameters of technological equipment and its operating modes, the state variables of all technological processes are random functions of space-time coordinates. In most cases, these accidents cannot be neglected, since they all affect the output parameters of the products. The most complex mathematical models of technological systems and processes in modern theory are random spatiotemporal fields, representing both input and output characteristics, as well as parameters of the systems under consideration. The purpose of this work is to model real-valued values of correlation functions of non-stationary random processes and sequences. When constructing a correlation theory of random processes and sequences, a complex representation is widely used, i.e. random functions of the form $\xi(t)=\xi_1(t)+i\xi_2(t), t-$ are considered: continuous or discrete time. This approach made it possible to construct a correlation theory of nonstationary random functions using the spectral theory of non-self-adjoint or unitary operators and to introduce the concept of complex spectrum. For applications of the correlation theory of nonstationary random functions and their modeling, it is convenient to deal with real-valued correlation functions. The construction of real-valued correlation functions can be carried out using the well-known fact that the real part of complex-valued correlation functions is also a correlation function (for the imaginary part this statement is unfair, since the imaginary part is a cross-correlation function of the real and imaginary parts of the corresponding random process or sequence) . The resulting models of correlation functions of non-stationary random processes and sequences can be used to construct algorithms for forecasting and filtering non-stationary random functions

Key words: mathematical expectation, correlation function, non-stationary random function, non-stationary random sequence, quasi-deterministic signals

Introduction.

Precision technological processes for the production of modern microelectronic products require compliance with the quality of source materials, working environments, and precise adherence to regimes. Due to natural fluctuations in the properties of materials and the environment, and variable states of all technological processes, the parameters of the product and technological processes cannot be described by deterministic laws. Due to the inevitable natural properties of fluctuations in the parameters of technological equipment and its operating modes, the state variables of all technological processes are random functions of space-time coordinates. In most cases, these contingencies cannot be neglected, since they all affect the output parameters of the products.

The most complex mathematical models of technological systems and processes in modern theory are random spatiotemporal fields, representing both input and output characteristics, as well as parameters of the systems under consideration. To solve complex technological problems, developments in modeling non-stationary random functions using correlation and poly-Gaussian methods can be used [4, 6, 7, 11, 12].

The purpose of this research is to model real values of correlation functions of non-stationary random processes and sequences.

Analysis of the latest research.

When constructing a correlation theory of random processes and sequences, a complex representation is widely used, i.e. random functions of the form are considered $\xi(t)=\xi_1(t)+i\xi_2(t), t$ – time is continuous or discrete [1-4, 7].

This approach made it possible to construct a correlation theory of nonstationary random functions using the spectral theory of non-self-adjoint or unitary operators and introduce the concept of complex spectrum [2]. The corresponding spectral expansions of nonstationary random functions represent, as in the stationary case, a superposition of internal states of harmonic (continuous or discrete oscillators), but with complex frequencies. New types of spectral expansions in internal states of strings (continuum harmonic oscillators) are appearing [2, 4]. For applications of the correlation theory of nonstationary random functions and their modeling, it is convenient to deal with real-valued correlation functions [7].

Formulation of the problem.

Therefore, the problem naturally arises of constructing real-valued correlation functions, the structure of which would take into account the complex spectrum. The problem of reconstructing nonstationary random functions from a given spectrum is effectively solved for fairly wide classes of nonstationary random functions by passing to the corresponding Hilbert space and using triangular and universal operator models [2]. In this case, it is essential that the corresponding Hilbert space is necessarily a complex Hilbert space, and, therefore, the correlation functions are complex-valued correlation functions.

The construction of real-valued correlation functions can be carried out using the well-known fact that the real part of complex-valued correlation functions is also a correlation function (for the imaginary part this statement is unfair, since the imaginary part is a mutual correlation function of the real and imaginary parts of the corresponding random process or sequence) [5]. The resulting models of correlation functions of non-stationary random processes and sequences can be used to construct algorithms for forecasting and filtering non-stationary random functions [8, 9].

Solution.

In the case of stationary random processes, the following correlation functions are very often used when analyzing experimental data [1]:

$$K(\tau)=Ce^{-\alpha|\tau|}\cos\beta\tau,$$
$$\left(C>0,\alpha>0,\tau=t-s,K(\tau)=M\xi(t)\overline{\xi(s)},\beta\in R\right);$$
$$K(\tau)=\frac{A}{4\alpha\omega^2}e^{-\alpha|\tau|}\left(\cos\beta\tau+\frac{\alpha}{\beta}\sin\beta\tau\right),$$
$$\left(\omega^2-\alpha^2=\beta^2>0\right);$$
$$K(\tau)=\frac{A}{4\alpha^3}e^{-\alpha|\tau|}\left(1+\alpha|\tau|\right),\left(\omega^2=\alpha^2\right);$$
$$K(\tau)=\frac{A}{8\alpha\beta_1\omega^2}\left(\left(\alpha+\beta_1\right)e^{-\alpha(\alpha-\beta_1)|\tau|}+\left(\alpha-\beta_1\right)e^{-\alpha(\alpha+\beta_1)|\tau|}\right),$$
$$\left(\omega^2-\alpha^2=-\beta^2<0,\beta=i\beta_1\right).$$

For stationary random sequences, correspondingly [1]:

$$K(0)=1,K(\tau)=0\ (\tau=m-n\neq 0);$$
$$K(\tau)=Ca^{|\tau|}\left(C>0,|a|<1,\tau=0,\pm1,\pm2,\ldots\right);$$
$$K(\tau)=\begin{cases}\dfrac{C(a-b)(1-ab)}{1-a^2}a^{|\tau|-1},\tau\neq 0,\\ \dfrac{C\left(1-2ab+b^2\right)}{1-a^2},\tau=0,\\ C>0,\ a,b\in R,\quad |a|<1,|b|<1,\end{cases}$$
$$K(\tau)=\frac{C}{\left(a_1-a_2\right)\left(1-a_1a_2\right)}\left(\frac{a_1}{1-a_1^2}a_1^{|\tau|}-\frac{a_2}{1-a_2^2}a_2^{|\tau|}\right),$$
$$\left(a_1\neq a_2\in R,\quad |a_1|<1,\quad |a_2|<1,\quad C\in R\right).$$

In the theory of nonstationary random processes and sequences [2], the following representations are obtained for the correlation functions of nonstationary random processes and sequences of finite nonstationarity rank, which in the corresponding Hilbert space are generated by dissipative operators (compression).

$$K(t,s)=K_\infty(t-s)+\int_0^\infty W(t+\tau,s+\tau)\,d\tau,\quad (t,s>0)\tag{1}$$

$$\left(K(n,m)=K_\infty(n-m)+\sum_{\tau=0}^{\infty}W(n+\tau,m+\tau)\right)$$

(in the case of asymptotic decay of a random function $K_\infty=0$).

For nonstationary random processes and sequences of finite rank (quasi-rank) of nonstationarity, representations for the functions were obtained in [2, 4]

$$W(t,s),W(n,m),V(t,s),V(n,m),$$

characterizing the deviation of the process (sequence) from the stationary (Hankel) one.

Infinitesimal correlation function $W(t,s)=-\dfrac{\partial}{\partial\tau}K(t+\tau,s+\tau)=\sum_{\alpha=1}^{r}\varphi_\alpha(t)\overline{\varphi_\alpha(s)},$

characterizing the deviation of a non-stationary random process from a stationary one. A $W(n,m)=K(n,m)-K(n+1,m+1)=\sum_{\alpha=1}^{r}\varphi_\alpha(n)\overline{\varphi_\alpha(m)}-$ correlation difference characterizing the deviation of a non-stationary random sequence from a stationary one.

Moreover,

$$K(t,s)=\left\langle\xi_t,\xi_s\right\rangle_{H_\xi},\quad \xi_t=e^{itA}\xi_0\xi_t=e^{itA}\xi_0,$$

$\left(K(n,m)=\left\langle\xi_n,\xi_m\right\rangle_{H_\xi},\quad \xi_n=T^n\xi_0\right)$ where $\xi_t\ (\xi_n)$ - is a nonstationary curve (sequence) in the corresponding Hilbert space H_ξ.

In the case when the subspaces $\overline{\dfrac{A-A^*}{i}H}$, $\left(\overline{\left(I-TT^*\right)H}\right)$ are finite-dimensional, we have

$$V(t,s)=-\frac{\partial}{\partial\tau}K(t+\tau,s+\tau)\Big|_{\tau=0}=i\sum_{\alpha=1}^{\rho}\varphi_\alpha(t)\overline{\varphi_\alpha(s)},$$
$$\varphi_\alpha(t)=\left\langle e^{itA}\xi_0,e_\alpha\right\rangle,\frac{T-T^*}{i}=\sum_{\alpha=1}^{r}\left\langle\circ,e_\alpha\right\rangle e_\alpha,$$
$$W(n,m)=K(n+1,m)-K(n,m+1)=i\sum_{\alpha=1}^{r}\varphi_\alpha(n)\overline{\varphi_\alpha(m)},$$
$$\varphi_\alpha(n)=\left\langle T^n\xi_0,e_\alpha\right\rangle,I-T^*T=\sum_{\alpha=1}^{\rho}\left\langle\circ,e_\alpha\right\rangle e_\alpha.$$

It follows that to obtain representations for real-valued correlation functions, it is enough to take the real part of (1) or the corresponding scalar product. Further $\operatorname{Re}K(t,s)$ we will denote $K_R(t,s),\operatorname{Re}W(t,s)$ accordingly $W_R(t,s)$. From (1) we get

$$K_R(t,s)=\operatorname{Re}K_\infty(t-s)+\int_0^\infty W(t+\tau,s+\tau)\,d\tau,\tag{2}$$

$$\left(K_R(n,m)=\operatorname{Re}K_\infty(n-m)+\sum_{\tau=0}^{\infty}W(n+\tau,m+\tau)\right)$$

or

$$K_R(t,s)=\operatorname{Re}K_\infty(t-s)+$$
$$+\int_0^\infty\sum_{\alpha=1}^{r}\left(x_\alpha(t+\tau)x_\alpha(s+\tau)+y_\alpha(t+\tau)y_\alpha(s+\tau)\right)d\tau.$$

Similar expressions have representations for $K(t,s)$, $K(n,m)$ in other cases.

Following representations will be obtained for the simplest non-stationary random processes and sequences. Consider a random sequence generated by a sequence in a Hilbert space $L^2_{[0;1]}$ of the form $\eta_n=T^n\eta_0$, where $T-$ a non-self-adjoint bounded operator of the form [2]

$$Tf(x)=\lambda_0 f(x)+i\int_0^1\varphi(x)\overline{\varphi(x)}f(y)dy,$$
$$\lambda_0=\alpha_0+i\beta_0\neq\overline{\lambda_0},\quad \left(\dim\overline{\operatorname{Im}TL^2_{[0;1]}}=\infty\right).$$

This sequence in a complex Hilbert space generates a real-valued correlation function of the form:

$$K_R(n,m)=r_0^{n+m}\cos(n-m)\varphi_0\left\|\hat f_0(x)\right\|^2+$$
$$+r_1^{n+m}\cos(n-m)\varphi_1\left\|\hat f_1(x)\right\|^2+$$
$$+r_0^n r_1^m\left(a\cos(n\varphi_0-m\varphi_1)+b\sin(m\varphi_1-n\varphi_0)\right)+$$
$$+r_1^n r_0^m\left(a\cos(n\varphi_1-m\varphi_0)+b\sin(n\varphi_1-m\varphi_0)\right),$$

were

$$r_0=\sqrt{\alpha_0^2+\beta_0^2},\ r_0<1,\ \varphi_0=\operatorname{arctg}\frac{\beta_0}{\alpha_0},$$
$$\hat f_0(x)=f_0(x)-\frac{\alpha_0\varphi(x)}{\gamma},$$
$$\gamma=\int_0^1\varphi(x)\overline{\varphi(x)}dx,\ a=\operatorname{Re}\left\langle\hat f_0(x),\hat f_1(x)\right\rangle,$$
$$b=\operatorname{Im}\left\langle\hat f_0(x),\hat f_1(x)\right\rangle.$$

Let us consider a more general case of a nonstationary random process in $L^2_{[0;1]}$ of the form $\xi_t=e^{itA}\xi_0$, where

$$Af(x)=a(x)f(x)+i\int_0^1\varphi(x)\overline{\theta(x)}f(y)dy,\ a(x)=\overline{a(x)}.$$

Then

$$u(x,t)=e^{ita(x)}f_0(x)-\varphi(x)\int_0^t e^{i(t-s)a(x)}\varphi(x)\gamma(s)ds,$$

where $\gamma(s)$ is the solution to the Volterra integral equation

$$\gamma(t)=\gamma_0(t)-\int_0^t G(t-s)\gamma(s)ds,$$
$$\gamma_0(t)=\int_0^t e^{ia(x)}\varphi(x)\overline{\theta(x)}dx,\ G(t-s)=\int_0^1 e^{i(t-s)a(x)}\varphi(x)\overline{\theta(x)}dx$$

In particular,

а) if $a(x)=-x,\ \varphi(x)=x,\ \theta(x)=\overline{\theta(x)}=1$,

$$G(t-s)=\int_0^1 xe^{-i(t-s)x}dx=\frac{i}{t-s}e^{-i(t-s)}+\frac{1}{(t-s)^2}e^{-i(t-s)}-\frac{1}{(t-s)^2},$$

б) if $a(x)=x,\ \varphi(x)=x,\ \theta(x)=\overline{\theta(x)}=1-x$,

$$G(t-s)=\int_0^1 x(1-x)e^{-i(t-s)x}dx=\frac{2\left(e^{i(t-s)}-1\right)}{i(t-s)^3}-\frac{1}{(t-s)^2}e^{i(t-s)}-$$
$$-\frac{1}{(t-s)^2},$$

в) if $a(x)=x,\ \varphi(x)=x,\ \theta(x)=\overline{\theta(x)}=1-x^2$,

$$G(t-s)=\int_0^1 x\left(1-x^2\right)e^{-i(t-s)x}dx=\frac{-1-2e^{i(t-s)}}{(t-s)^2}+$$
$$+\frac{6e^{i(t-s)}}{i(t-s)^3}+\frac{6\left(e^{i(t-s)}-1\right)}{(t-s)^4}.$$

Let's consider $u(x,t)=e^{itA}f_0(x)$, where $A-$ limited dissipative operator in l_2 of the form [2]:

$$(Af)_k=\lambda_k f_k+i\sum_{j=k+1}^{N}f(j)\beta_j\beta_k$$
$$\left(k=1,2,\ldots,N,\quad \lambda_k=\alpha_k+i\frac{\beta_k^2}{2}\right).$$

Then the correlation function has the form $K(t,s)=\int_0^\infty\varphi(t+\tau)\overline{\varphi(s+\tau)}d\tau$,

where $\varphi(t)=\sum_{k=1}^{N<\infty}C_k\overline{\Lambda_k(t)}$ and

$$\Lambda_k(t)=-\frac{1}{2\pi i}\oint_\gamma e^{i\lambda t}\frac{\beta_k}{\overline{\lambda_k}-\lambda}\prod_{j=1}^{k-1}\frac{\lambda_j-\lambda}{\overline{\lambda_j}-\lambda}d\lambda,\quad \gamma-$$

circuit covering the entire spectrum of the operator *A*.

$$\overline{\Lambda_1(t)}=\overline{-\frac{1}{2\pi i}\oint_\gamma e^{i\lambda t}\frac{\beta_1}{\overline{\lambda_1}-\lambda}d\lambda}=\beta_1 e^{i\lambda_1 t},$$
$$\overline{\Lambda_2(t)}=\overline{-\frac{1}{2\pi i}\oint_\gamma e^{i\lambda t}\frac{\beta_2}{\overline{\lambda_2}-\lambda}\cdot\frac{\lambda_1-\lambda}{\overline{\lambda_1}-\lambda}d\lambda}=\beta_2 e^{i\lambda_2 t}\frac{\overline{\lambda_1}-\lambda_2}{\lambda_1-\lambda_2}+$$
$$+e^{i\lambda_1 t}\frac{i\beta_2\beta_1^2}{\overline{\lambda_1}-\lambda_2},\lambda_1\neq\lambda_2,$$
$$\overline{\Lambda_3(t)}=\overline{-\frac{1}{2\pi i}\oint_\gamma e^{i\lambda t}\frac{\beta_3}{\overline{\lambda_3}-\lambda}\cdot\frac{\lambda_1-\lambda}{\overline{\lambda_1}-\lambda}\cdot\frac{\lambda_2-\lambda}{\overline{\lambda_2}-\lambda}d\lambda}=$$
$$=\beta_3 e^{i\lambda_3 t}\frac{\overline{\lambda_1}-\lambda_3}{\lambda_1-\lambda_3}+e^{i\lambda_2 t}\frac{i\beta_3\beta_2^2}{\overline{\lambda_2}-\lambda_3}\cdot\frac{\overline{\lambda_1}-\lambda_2}{\lambda_1-\lambda_2}+e^{i\lambda_1 t}\frac{i\beta_3\beta_1^2}{\overline{\lambda_1}-\lambda_3}\cdot\frac{\overline{\lambda_2}-\lambda_1}{\lambda_2-\lambda_1},$$
$$\lambda_1\neq\lambda_2\neq\lambda_3,$$

if $\lambda_1=\lambda_2\neq\lambda_3$,

Thus, for modeling real-valued correlation functions of the simplest non-stationary random processes (sequences), a simple algorithm is obtained:

We consider a complex Hilbert space $L_2(\Omega)$, which is a subspace of the Hilbert space; then we consider a curve (sequence) $\hat{\xi}_t = e^{itA}\xi_0$, $\left(\hat{\xi}_n = T^n\xi_0\right)$ in this space.

Then, for various classes of non-self-adjoint (non-unitary) operators *A* (*T*), the scalar product is calculated, which is the correlation function of the original ones ξ_t (ξ_n). When calculating the scalar product, you can go to unitarily equivalent elements, which allows you to use model (triangular, functional, universal) representations of the operator *A* (*T*). Calculation $\mathrm{Re}\left\langle \hat{\xi}_t, \hat{\xi}_s \right\rangle_{H_\xi}$ $\left(\mathrm{Re}\left\langle \hat{\xi}_n, \hat{\xi}_m \right\rangle_{H_\xi}\right)$ leads to the desired result.

Similar representations of real-valued correlation functions can be obtained for inhomogeneous continuous and discrete fields [6, 7, 11, 12].

The simplest forecast algorithm uses the values of a random function at one point in time $\xi(t_0)$.

Conclusions and directions for further research

The main results obtained in this article were used in the operation of technological quality assurance systems for microelectronics products. From the above relations it is easy to see that to represent the parameters of a real technological process in the form of a Gaussian random process (or random field), it is enough to select the average value and the correlation function at any given time (or at any point in space) and the correlation coefficients between the readings.

At the same time, a prerequisite for creating automatic process control systems is the construction of adequate models of all operations included in the controlled technological process and an exhaustively complete description of all disturbing influences. For example, for the chemical etching process, which is widely used in microelectronics technology, the equipment modes (etching temperature, etching type, bath or reactor size, and thickness and material of the etching layer) are selected in advance and remain unchanged during the operation. However, the etching rate, which largely determines the final result (material removal, minimal dimensional distortion, etching selectivity, etc.), changes due to changes over time in such influences as the concentration of etching particles at the surface of the solid phase, temperature changes due to heat release during reactions, features of the microtexture of the material, different kinetics of etching along the depth of the material.

If we consider the etching operation as a process occurring precisely on the surface of the solid phase, then all these variable influences can be considered external. In addition, over time, the parameters of technological equipment, feedstock and external conditions change. These changes are often random and lead to the so-called a priori insufficiency, that is, to a lack of information about the operating conditions of the control object. To overcome a priori insufficiency, they use the creation of adaptive, that is, automated control systems that adapt to changing conditions. A model of a nonstationary random process that displays the dependence of the etching wedge on the thickness of the photoresistive mask is considered in [10].

Список літератури

1. Яглом А.М. Введение в теорию стационарных случайных функций // УМН. 1952. Том 1. – Вип.5(51),– С. 3–168.
2. Лившиц М.С., Янцевич А.А. Теория операторных узлов в гильбертовых пространствах / Х: ХГУ, 1971. – 160с.
3. Свешников А.А. Прикладные методы теории случайных функций // М.: Наука, 1968. – 463с.
4. Черемская Н.В. О моделировании некоторых классов нестационарных случайных последовательностей при помощи треугольных моделей операторов // Радиотехника. 2003. – Вип. 132. – С.70–77.
5. Лоэв М. Теория вероятностей / М.: Изд-во иностр. лит-ры. 1962. – 719с.
6. Черемская Н.В. Последовательности в гильбертовом пространстве, определяемые уравнениями в частных разностях // Вісник Харківського університету, сер. Математика, прикладна математика і механіка. 2000. – №475. – С. 366–374.
7. Черемська Н.В. Моделювання дійснозначних кореляційних функцій з урахуванням комплексного спектру. //Вісник НТУ «ХПІ». Серія: Математичне моделювання в техніці та технологіях. № 6 (1228) 2017.
8. Cheremskaya N. V. Developing algorithms of optimal forecasting and filtering for some classes of nonstationary random sequences // Вісник НТУ «ХПІ». Серія: Математичне моделювання в техниці та технологіях. – Харків: НТУ «ХПІ», 2018. – № 3 (1279) 2018. – С. 139–145.
9. Cheremskaya N. V. Dependence of prognosis and filtration failure on different values of parameters for some classes of non-stationary random sequences // Вісник НТУ «ХПІ». Серія: Математичне моделювання в техніці та технологіях. № 8 (1333) 2019.
10. Антонова В.А., Петрова А.Ю., Черемська Н.В., Янцевич А.А. Застосування моделей нестаціонарних випадкових процесів до операції хімічного рідинного травління в технологічних процесах мікроелекторніки // Сборник научных трудов 1-й Международной конференции «Электоронная компонентная база. Состояние и перспективы развития».– Харьков – Судак, 2008. –Том III. – С. 116-119.
11. Петрова А.Ю. Восстановление случайных полей по корреляционным функціям // Вестник НТУ «ХПИ». Сб. науч. тр. Вып.. «Системный анализ, управление и информационные технологии». – Х., НТУ «ХПИ», 2003.– №6. – Т.1. – С. 174-182.
12. Петрова А.Ю. Корреляционная теория некоторых классов нестационарных случайных функций конечного ранга нестационарности // Радиоэлектроника и інформатика.- 2007. - №1. - С.29-34.

References (transliterated)

1. Yaglom A.M. Vvedeniye v teoriyu statsionarnykh sluchaynykh funktsiy // UMN. – 1952. –Tom 1. – Vyp.5(51),– S.3-168.
2. Livshits M.S., Yantsevich A.A. Teoriya operatornykh uzlov v gil'bertovykh prostranstvakh / Khar'kov: Izd-vo KHGU, 1971. – 160s.

3. Sveshnikov A.A. Prikladnyye metody teorii sluchaynykh funktsiy // M.:Nauka, 1968. 463 p.
4. Cheremskaya N.V. O modelirovanii nekotorykh klassov nestatsionarnykh sluchaynykh posledovatel'nostey pri pomoshchi treugol'nykh modeley operatorov // Radiotekhnika. – 2003. – Vyp. 132. – Pp.70-77.
5. Loev M. Teoriya veroyatnostey / M.: Izd-vo inostr. lit-ry – 1962. – 719 p.
6. Cheremskaya N.V. Posledovatel'nosti v gil'bertovom prostranstve, opredelyayemyye uravneniyami v chastnykh raznostyakh // Vísnik Kharkívs'kogo uníversitetu, ser. Matematika, prikladna matematika í mekhaníka. – 2000. – 475. – Pp. 366-374.
7. Cherems'ka N.V. Modelyuvannya díysnoznachnikh korelyatsíynikh funktsíy z urakhuvannyam kompleksnogo spektru. //Vísnik NTU «KhPÍ». Seríya: Matematichne modelyuvannya v tekhnítsí ta tekhnologíyakh. № 6 (1228) 2017.
8. Cheremskaya N. V. Developing algorithms of optimal forecasting and filtering for some classes of nonstationary random sequences // Vísnik NTU «KhPÍ». Seríya: Matematichne modelyuvannya v tekhnitsí ta tekhnologíyakh. – Kharkív: NTU «KhPÍ», 2018. – № 3 (1279) 2018. – Pp. 139–145.
9. Cheremskaya N. V. Dependence of prognosis and filtration failure on different values of parameters for some classes of non-stationary random sequences // Vísnik NTU «KhPÍ». Seríya: Matematichne modelyuvannya v tekhnítsí ta tekhnologíyakh. № 8 (1333) 2019.
10. Antonova V.A., Petrova A.YU., Cherems'ka N.V., Yantsevich A.A. Zastosuvannya modeley nestatsíonarnikh vipadkovikh protsesív do operatsíї khímíchnogo rídinnogo travlínnya v tekhnologíchnikh protsesakh míkroyelektorníki // Sbornik nauchnykh trudov 1-y Mezhdunarodnoy konferentsii «Elektoronnaya komponentnaya baza. Sostoyaniye i perspektivy razvitiya».– Khar'kiv – Sudak, 2008. –Tom III. – Pp. 116-119.
11. Petrova A.YU. Vosstanovleniye sluchaynykh poley po korrelyatsionnym funktsíyam // Vestnik NTU «KhPI». Sb. nauch. tr. Vyp.. «Sistemnyy analiz, upravleniye i informatsionnyye tekhnologii». – Kh., NTU «KhPI», 2003.– №6. – T.1. – Pp. 174-182.
12. Petrova A.YU. Korrelyatsionnaya teoriya nekotorykh klassov nestatsionarnykh sluchaynykh funktsiy konechnogo ranga nestatsionarnosti // Radioelektronika i ínformatika. 2007. №1. Pp.29-34

Надійшла (received) 19.06.2024

Відомості про авторів / Сведения об авторах / About the Authors

Черемська Надія Валентинівна (Черемская Надежда Валентиновна, Cheremska Nadezhda Valentinovna) – кандидат технічних наук, доцент, Національний технічний університет «Харківський політехнічний інститут», м. Харків, Україна; тел.: (050) 225-15-44; e-mail: cheremskaya66@gmail.com.

Н. В. ЧЕРЕМСЬКА

МОДЕЛІ КОРЕЛЯЦІЙНИХ ФУНКЦІЙ НЕСТАЦІОНАРНИХ ПРОЦЕСІВ ТА ПОСЛІДОВНОСТЕЙ ДЛЯ ТЕХНОЛОГІЧНИХ СИСТЕМ

Прецизійні технологічні процеси виробництва сучасних мікроелектронних виробів потребують дотримання якості вихідних матеріалів, робочих середовищ, точності дотримання режимів. Параметри виробу та технологічних процесів через природні флуктуації властивостей матеріалів та навколишнього середовища, змінних станів усіх технологічних процесів не можуть бути описані детермінованими закономірностями. Через неминучі природні властивості флуктуацій параметрів технологічного обладнання та режимів його роботи змінні стани всіх технологічних процесів є випадковими функціями просторово-часових координат. Найчастіше цими випадковостями знехтувати не вдається, оскільки вони впливають на вихідні параметри виробів. Найбільш складними в сучасній теорії математичними моделями технологічних систем і процесів є випадкові просторово-часові поля, що представляють як вхідні та вихідні характеристики, так і параметри систем, що розглядаються. Метою даної є моделювання дійснозначних значень кореляційних функцій нестаціонарних випадкових процесів і послідовностей. При побудові кореляційної теорії випадкових процесів і послідовностей широко використовується комплексне уявлення, тобто розглядаються випадкові функції виду: $\xi(t)=\xi_1(t)+i\xi_2(t), t-$ час неперервний або дискретний. Такий підхід дозволив побудувати кореляційну теорію випадкових нестаціонарних функцій за допомогою спектральної теорії несамоспряжених або унітарних операторів і ввести поняття комплексного спектру. Для застосувань кореляційної теорії нестаціонарних випадкових функцій та їх моделювання зручно мати справу із дійснозначними кореляційними функціями. Побудову дійснозначних кореляційних функцій можна здійснити, використовуючи той відомий факт, що дійсна частина комплекснозначних кореляційних функцій також є кореляційною функцією (для уявної частини це твердження несправедливо, тому що уявна частина є взаємною кореляційною функцією дійсної та уявної частин) . Отримані моделі кореляційних функцій випадкових нестаціонарних процесів і послідовностей можуть бути використані для побудови алгоритмів прогнозу і фільтрації нестаціонарних випадкових функцій.

Ключові слова: кореляційна функція, нестаціонарна випадкова функція, нестаціонарна випадкова послідовність, спектральна теорія не самоспряжених або унітарних операторів, кореляційна різниця